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Timothy D. Townsend

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A KEY

TO THE

AMERICAN EDITION

OF

John

BONNYCASTLE'S MENSURATION.

CONTAINING

Solutions to all the Questions left unsolved in that Work.

BY BENJAMIN HALLOWELL.

NEW EDITION.

Adapted to the revised edition of the Mensuration,

BY JAMES RYAN.

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ADVERTISEMENT.

The following Key, being designed not only to assist those who are pursuing their studies without an instructer, but to abridge the labour of teachers, by furnishing them with the means of reference, care has been taken to illustrate the solutions by referring, when necessary, to appropriate rules and observations in the Mensuration, and so to exhibit the results of the different operations, as to make it easy to detect errors in the work of a pupil.

It will be observed that quantities are frequently used in the form of vulgar fractions instead of decimals, as being shorter and more accurate.

In the solutions of some questions of a geometrical nature, a few references are made to Playfair's Geometry.

B. H.

A KEY

TO

BONNYCASTLE'S MENSURATION.

MENSURATION OF SUPERFICIES.

PROBLEM I.

PAGE 51

To find the area of a parallelogram.

Ex. 5. Here $35.25 \times 35.25 = 1242.5625$ *sq. ch.* and $1242.5625 \div 10 = 124.25625$ *ac.* $= 124$ *ac.* 1 *r.* 1 *po.* Ans.

Ex. 6. Here 8 *ft.* 4 *in.* $= 8\frac{1}{3}$ *ft.* $= \frac{25}{3}$ *ft.*

And $\frac{25}{3} \times \frac{25}{3} = \frac{625}{9} = 69$ *ft.* 5 *in.* 4 *pa.* Ans.

Ex. 7. Here 14 *ft.* 6 *in.* $= 14.5$ *ft.* and 4 *ft.* 9 *in.* $=$ 4.75 *ft.*

Whence $14.5 \times 4.75 = 68.875$ *ft.* $= 68$ *ft.* 10 *in.* 6 *pa.* Ans.

Ex. 8. Here $12.24 \times 9.16 = 112.1184$ *ft.* $= 112$ *ft.* 1 *in.* 5 *pa.* Ans.

Ex. 9. Here $10.51 \times 4.28 = 44.9828$ *sq. ch.* $= 4$ *ac.* 1 *ro.* 39.7248 *per.* Ans.

Ex. 10. Here 7 *ft.* 9 *in.* $=7.75$ *ft.* and 3 *ft.* 6 *in.* $=3.5$ *ft.*
Whence $7.75\times 3.5=27.125$ *sq. ft.* $=27$ *ft.* 1 *in.* 6 *pa.* Ans.

Ex. 11. Here $12\frac{1}{2}=\frac{25}{2}$ *ft.* and 9 *in.* $=\frac{3}{4}$ *ft.*
Whence $\frac{25}{2}\times\frac{3}{4}=\frac{75}{8}=9\frac{3}{8}$ *ft.* Ans.

PROBLEM II.

PAGE 54.

To find the area of a triangle, having the base and perpendicular height given.

Ex. 2. Here 18 *ft.* 4 *in.* $=18\frac{1}{3}$ *ft.* $=\frac{55}{3}$ and 11 *ft.* 10 *in.* $=11\frac{5}{6}$ *ft.* $=\frac{71}{6}$ *ft.*

Whence $\frac{55}{3}\times\frac{71}{6}=\frac{3905}{18}$ and this divided by $2=\frac{3905}{36}=$ 108 *ft.* 5 *in.* 8 *pa.* Ans.

Ex. 3. Here $\dfrac{16.75\times 6.24}{2}=\dfrac{104.52}{2}=52.26$ *ft.* $=52$ *ft.* 3 *in.* 1.44 *pa.* Ans.

Ex. 4. Here $\dfrac{12.25\times 8.5}{2}=\dfrac{104.125}{2}=52.0625$ *sq. ch.* $=5$ *ac.* 0 *ro.* 33 *per.* Ans.

Ex. 5. Here $\dfrac{20\times 10.25}{2}=\dfrac{205}{2}=102.5$ *ft.* Ans.

PROBLEM III.

PAGE 55.

To find the area of a triangle whose three sides are given.

Ex. 2. Here $\dfrac{13+14+15}{2}=\dfrac{42}{2}=21=$ half the sum of the sides. Also $21-13=8=$ first difference, $21-14=7=$ 2d difference, and $21-15=6=$ 3d difference.

Hence $\sqrt{21\times 8\times 7\times 6}=\sqrt{7056}=84=$ area required.

Ex. 3. Here $\frac{49+50.25+25.69}{2}=\frac{124.94}{2}=62.47=$half the sum of the sides, hence the differences will be respectively 13.47, 12.22, and 36.78.

Wherefore $\sqrt{62.47\times13.47\times12.22\times36.78}=$*$\sqrt{841.4709\times449.4516}=\sqrt{378200.44235844}=614.98$ *sq.ch*$=61.498$ *acres.* Ans.

Ex. 4. Here $\frac{50+40+30}{2}=\frac{120}{2}=60=$half the sum of the sides, hence the differences will be 10, 20, and 30, respectively.

Wherefore $\sqrt{60\times10\times20\times30}=\sqrt{360000}=600$. Ans.

Note.—This triangle being right-angled, the area may be found more concisely by the last Problem, thus: $\frac{40\times30}{2}=600$. Ans.

Ex. 5. Here $\frac{25+25+25}{2}=\frac{75}{2}=37.5=$ half the sum of the sides. Hence each difference is 12.5; wherefore $\sqrt{37.5\times12.5\times12.5\times12.5}=\sqrt{73242.1875}=270.6329$. Ans.

Ex. 6. Here $\frac{20+15+15}{2}=\frac{50}{2}=25=$half the sum of the three sides, hence the differences are 5, 10 and 10 respectively.

Wherefore $\sqrt{25\times5\times10\times10}=\sqrt{12500}=111.8034$. Ans.

Ex. 7. Here $\frac{20+30+40}{2}=45=$half sum of the sides, hence the differences are 25, 15, and 5.

Wherefore $\sqrt{45\times25\times15\times5}=\sqrt{84375}=290.4738$ *sq. ch.*$=29$ *ac.* 0 *ro.* 7.58 *po.* Ans.

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* Here, as in some other places, the first two numbers are multiplied together, and also the other two numbers, and then these two products, in order the more readily to detect an error.

PROBLEM IV.

PAGE 57.

Any two sides of a right-angled triangle being given to find the third.

Ex. 3. Here $\sqrt{77^2+36^2}=\sqrt{7225}=85$. Ans.

Ex. 4. Here $\sqrt{109^2-60^2}=\sqrt{8281}=91$. Ans.

Ex. 5. Here $\sqrt{20^2+12^2}=\sqrt{544}=23.3238$. Ans

Ex. 6. Here $\sqrt{320^2-103^2}=\sqrt{102400-10609}=\sqrt{91791}=302.9703$. Ans.

Ex. 7. Here we must find the bases of two right-angled triangles, the hypothenuse of each being 50, and the perpendiculars 28 and 36 respectively: the sum of these bases will be the width required: thus,
$\sqrt{50^2-28^2}=\sqrt{1716}=41.4246303$=one part,
and $\sqrt{50^2-36^2}=\sqrt{1204}=34.6987032$= the other part, their sum=76.1233335=the width required.

PROBLEM V.

PAGE 59.

To find the area of a trapezium.

Ex. 2. Here $24.5+30.1=54.6$=the sum of the perpendiculars.

And $\frac{54.6\times 80.5}{2}=\frac{4395.30}{2}=2197.65$. Ans.

Ex. 3. Here 60*ft.* 9 *in.*+56*ft.* 3 *in.*=117*ft.*=the sum of the perpendiculars.

And $\frac{117\times 108.5}{2}=\frac{12694.5}{2}=6347.25$*ft.*=6347*ft.* 3 *in.*

Ans.

PROBLEM VI.

PAGE 60.

To find the area of a trapezoid.

Ex. 2. Here $12.41+8.22=20.63=$ the sum of the parallel sides,

And $\frac{20.63\times5.15}{2}=\frac{106.2445}{2}=53.12225$ *sq. ch.*$=5$ *ac.* 1 *ro.* 9.956 *po.* Ans.

Ex. 3. Here 25 *ft.* 6 *in.* + 18 *ft.* 9 *in.* = 44 *ft.* 3 *in.* = the sum of the parallel sides,

And 44 *ft.* 3 *in.* × 10 *ft.* 5 *in.* = 460 *ft.* 11 *in.* 3 *pa.*, which, divided by 2, will give 230 *ft.* 5 *in.* 7.5 *pa.* Ans.

Ex. 4. Here $20.5+12.25=32.75=$ sum of the parallel sides,

And $\frac{32.75\times10.75}{2}=\frac{352.0625}{2}=176.03125$. Ans.

PROBLEM VII.

PAGE 61.

To find the area of a regular polygon.

Ex. 2. Here $\frac{14.6\times6}{2}=\frac{87.6}{2}=43.8=$ half the perimeter,

And $43.8\times12.64=553.632$ *ft.* Ans.

Ex. 3. Here $\frac{19.38\times7}{2}=\frac{135.66}{2}=67.83=$ half perimeter,

And $67.83\times20=1356.60$. Ans.

Ex. 4. Here $\frac{9.941\times8}{2}=39.764=$ half perimeter,

And $39.764\times12=477.168$. Ans.

PROBLEM VIII.

PAGE 62.

To find the area of a regular polygon when the side only is given.

Ex. 2. Here 5 *ft.* 4 *in.* $=5\frac{1}{3}$ *ft.* $=\frac{16}{3}$,

And $2.598076\times\left(\frac{16}{3}\right)^2=2.598076\times\frac{256}{9}=\frac{665.107456}{9}$ $=73.9008$. Ans.

Ex. 3. Here $4.828427\times 16^2=4.828427\times 256=1236.077312$. Ans.

Ex. 4. Here $7.694209\times 20.5^2=7.694209\times 420.25=3233.49133225$. Ans.

Ex. 5. Here $6.181824\times 36^2=6.181824\times 1296=8011.643904$. Ans.

Ex. 6. Here $11.196152\times 125^2=11.196152\times 15625=174939.875$. Ans.

PROBLEM IX.

PAGE 64.

The diameter of a circle being given to find the circumference, or the circumference of a circle being given to find the diameter.

Ex. 3. Here $3.1416\times 40=125.6640=$ the circumference.

Ex. 4. Here $3.1416\times 12=37.6992=$ the circumference.

Ex. 5. Here $25000\div 3.1416=7958$ nearly. Ans.

Ex. 6. Here $64\div 3.1416=20.3718$. Ans.

PROBLEM X.

PAGE 67.

To find the length of any arc of a circle.

Ex. 3. Here $\overline{7^2 \times 15} + \overline{2^2 \times 33} = 867$, reserved number,

And $\sqrt{\overline{7^2 + 2^2 \times 4}} = \sqrt{65} = 8.0623$ = twice the chord of half the arc,

Also $\dfrac{8.0623 \times 2^2 \times 10}{867} = \dfrac{322.4920}{867} = .3720$, which, added to twice the chord of half the arc, will make 8.4343 = the length required.

Ex. 4. Here $\overline{40^2 \times 15} + \overline{15^2 \times 33} = 31425$ reserved number,

And $\sqrt{\overline{40^2 + 15^2 \times 4}} = \sqrt{2500} = 50$ = twice the chord of half the arc,

Also $\dfrac{50 \times 15^2 \times 10}{31425} = \dfrac{112500}{31425} = 3.5800$; which, added to twice the chord of half the arc (50), will make 53.5800. Ans.

Ex. 5. Here $27^2 - 25^2 = 729 - 625 = 104$ = square of the versed sine,

And $\overline{50^2 \times 15} + \overline{104 \times 33} = 40932$ reserved number,

Also $27 \times 2 = 54$ twice the chord of half the arc,

Hence $\dfrac{54 \times 104 \times 10}{40932} = \dfrac{56160}{40932} = 1.3720$, which, added to twice the chord of half the arc (54), will make 55.3720. Ans.

Ex. 6. Here $\overline{100 \times 60} - \overline{9 \times 27} = 5757$ reserved number,

And $\sqrt{\overline{100 \times 9}} = \sqrt{900} = 30$ = chord of half the arc,

Also $\dfrac{\overline{30 \times 2} \times 9 \times 10}{5757} = \dfrac{5400}{5757} = .9380$, which, added to twice the chord of half the arc (60), will make 60.9380, the length required.

Ex. 7. By note 1, page 68, the versed sine may be found as follows:

$\frac{20-\sqrt{20^2-16^2}}{2}=\frac{20-\sqrt{144}}{2}=\frac{20-12}{2}=4$, the versed sine,

Then by the second part of the rule $\overline{20\times 60}-\overline{4\times 27}=$ 1092 reserved number,

And $\sqrt{20\times 4}=\sqrt{80}=8.9443=$chord of half the arc,

Also $\frac{\overline{8.9443\times 2}\times\overline{4\times 10}}{1092}=\frac{715.5440}{1092}=.6553$ which, added to twice the chord of half the arc, (17.8886) will make 18.5439. Ans.

Ex. 8. By note second, page 68, the versed sine$=\frac{30^2}{50}$ $=\frac{900}{50}=18$,

And by the second part of the rule $\overline{50\times 60}-\overline{18\times 27}=$ $2514=$reserved number.

Also $\frac{\overline{30\times 2}\times\overline{18\times 10}}{2514}=\frac{10800}{2514}=4.2959$, which, added to twice the chord of half the arc (60), will make 64.2959. Ans.

Ex. 9. Here, by note second, page 68, the diameter$=$ $\frac{25^2}{15}=\frac{625}{15}=41\frac{2}{3}$.

And by the second part of the rule $\overline{41\frac{2}{3}\times 60}-\overline{15\times 27}$ $=2095=$reserved number.

Also $\frac{\overline{25\times 2}\times\overline{15\times 10}}{2095}=\frac{7500}{2095}=3.5800$, which, added to twice the chord of half the arc (50), will make $53.58=$the length required.

PROBLEM XI.

To find the area of a circle.

RULE I.—PAGE 70.

Ex. 3. Here $\frac{3.1416}{2} \times \frac{1}{2} = .7854$. Ans.

Ex. 4. Here $\frac{22}{2} \times \frac{7}{2} = \frac{154}{4} = 38\frac{1}{2}$. Ans.

RULE II.—PAGE 72.

Ex. 2. Here $7^2 \times .7854 = 49 \times .7854 = 38.4846$. Ans.

Ex. 3. Here $\overline{4.5}|^2 \times .7854 = 20.25 \times .7854 = 15.90435$. Ans.

Ex. 4. Here $\overline{3.5}|^2 \times .7854 = 12.25 \times .7854 = 9.62115$ *sq. ft.* $= 1.0690$ *sq. yds.* Ans.

Ex. 5. Here $10.9956^2 \times .07958 = 120.90321936 \times .07958 = 9.6214781966688$ *square yards*, which, reduced to square feet, will make 86.5933. Ans.

Ex. 6. Here 7 *miles* = 2240 *per.* and $2240^2 \times .07958 = 5017600 \times .07958 = 399300.6080$. Ans.

PROBLEM XII.

To find the area of a sector of a circle.

RULE I.—PAGE 73.

Ex. 2. In the 4th example to Problem X. we have found the length of this arc to be 53.5800; and by the note, page 74, the radius $= \frac{20^2+15^2}{15 \times 2} = \frac{625}{30} = 20\frac{5}{6}$.

Hence $(53.5800 \times 20\frac{5}{6}) \div 2 = \frac{1116.25}{2} = 558.1250 =$ the area of the sector required.

Ex. 3. By note second, page 68, the versed sine $=\frac{30^2}{100}$ $=\frac{900}{100}=9$; and by the second part of the rule to Problem X. $\overline{100\times 60}-\overline{9\times 27}=5757=$ reserved number.

Also $\frac{\overline{30\times 2}\times\overline{9\times 10}}{5757}=\frac{5400}{5757}=.9380$, which, added to twice the chord of half the arc (60), will make $60.9380=$ the length of the arc.

Hence $\frac{60.9380\times 50}{2}=\frac{3046.9000}{2}=1523.4600=$ the area of the sector required.

Ex. 4. Here $\overline{50\times 60}-\overline{18\times 27}=2514=$ reserved number, And $\sqrt{\overline{18\times 50}}=\sqrt{\overline{900}}=30=$ chord of half the arc.

Also $\frac{\overline{30\times 2}\times\overline{18\times 10}}{2514}=\frac{10800}{2514}=4.2959$, which, added to twice the chord of half the arc (60), will make 64.2959 $=$ the length of the arc.

Hence $\frac{64.2959\times 25}{2}=\frac{1607.3975}{2}=803.69875=$ the area of the sector required.

RULE II.—PAGE 75.

Ex. 2. Here $50^2\times .7854=2500\times .7854=1963.5000=$ the area of the circle.

And, as $360^\circ : 147^\circ\ 29' :: 1963.5 : 804.3987=$ area of the sector.

Ex. 3. Here $26^2\times .7854=676\times .7854=530.9304=$ area of the circle,

And $\frac{530.9304}{2}=265.4652=$ the area of the semi-circle.

Ex. 4. Here $42^2\times .7854=1764\times .7854=1385.4456=$ area of the circle,

And $\frac{1385.4456}{4}=346.3614=$ area of the quadrant.

PROBLEM XIII.

To find the area of a segment of a circle.

RULE I.—PAGE 76.

Ex. 2. Here $\overline{20^2\times15}+\overline{2^2\times33}=6132$, reserved number,

And $\sqrt{20^2+\overline{2^2\times4}}=\sqrt{416}=20.3961$=twice the chord of half the arc.

Also $\frac{20.3961\times\overline{2^2\times10}}{6132}=\frac{815.8440}{6132}=.1330$, which, added to twice the chord of half the arc, will make 20.5291= the length of the arc.

By note, page 74, the radius$=\frac{10^2+2^2}{2\times2}=\frac{104}{4}=26$.

Hence $\frac{20.5291\times26}{2}=\frac{533.7566}{2}=266.8783$, area of the sector.

By note, page 76, $\frac{20}{2}\times\overline{26-2}=10\times24=240$=area of the triangle.

Wherefore, 266.8783—240=26.8783=area of the segment required.

Ex. 3. By note 1st, page 68, the versed sine = $\frac{20-\sqrt{20^2-12^2}}{2}=\frac{20-16}{2}=2$,

And $\overline{20\times60}-\overline{2\times27}=1146$, reserved number.

Also $\sqrt{20\times2}=\sqrt{40}=6.3246$=chord of half the arc.

Then $\frac{\overline{6.3246\times2}\times\overline{2\times10}}{1146}=\frac{252.9840}{1146}=.2208$, which, added to twice the chord of half the arc (12.6492), will make 12.8700=the length of the arc.

Hence $\frac{12.8700\times10}{2}=64.3500$=area of the sector.

By note, page 76, $\frac{12}{2} \times \overline{10-2} = 6 \times 8 = 48 =$ area of the triangle,

Wherefore $64.3500 - 48 = 16.3500 =$ area of the segment required.

Ex. 4. In Problem X. Example 7, we have found the length of this arc to be 18.5439, and the versed sine 4.

Hence $\frac{18.5439 \times 10}{2} = 92.7195 =$ area of the segment.

Also $\frac{16}{2} \times \overline{10-4} = 8 \times 6 = 48 =$ area of the triangle.

Wherefore $92.7195 - 48 = 44.7195 =$ area of the seg.

Ex. 5. Here $\frac{18^2 \times .7854}{} = \frac{324 \times .7854}{4} = 63.6174 =$ area of the sector.

And $\frac{9 \times 9}{2} = \frac{81}{2} = 40.5 =$ the area of the triangle.

Hence $63.6174 - 40.5 = 23.1174$, area of the segment.

Ex. 6. Here $50^2 \times .7854 = 2500 \times .7854 = 1963.5 =$ the area of the circle,

And $360° : 300° :: 1963.5 : 1636.25 =$ area of the sector.

Now, it is evident that the triangle will be equilateral, having each side equal to the radius $= 25$; hence, by Problem VIII. of Superficies, $.433013 \times 25^2 = 270.6331 =$ the area of the triangle,

Wherefore $1636.25 + 270.6331 = 1906.8831 =$ area of the segment.

RULE II.—PAGE 78.

Ex. 2. Here $\frac{5}{25} = .200 =$ tabular versed sine, the segment corresponding to which is .111823.

Now $.111823 \times 25^2 = .111823 \times 625 = 69.889375$. Ans.

Ex. 3. Here $\frac{10}{40}=.250$ = tabular versed sine, the segment corresponding to which is .153546.

Now $.153546 \times 40^2 = 245.6736$. Ans.

Ex. 4. Here $\frac{2}{52}=.0385$ = tabular versed sine.

Tabular segment corresponding to .038 is .009763,
Do. do. do. .039 .010148,
Difference .000385,

Now $.000385 \times .5 = .0001925$, which, added to the segment corresponding to .038, will give the segment corresponding to $.0385 = .0099555$.

Hence $.0099555 \times 52^2 = 26.919672$ = area of the seg.

Ex. 5. Here $\frac{9}{100}=.090$ = tabular versed sine, the segment corresponding to which is .035011; and this, multiplied by 100^2, will produce 350.1100 = the area of the segment.

Ex. 6. Here $\sqrt{(50^2-30^2)} = \sqrt{1600} = 40$; which is the difference between the radius and versed sine:

Therefore $50-40=10$, the versed sine.

Hence $\frac{10}{100}=.1$, the tabular versed sine,

And the tabular segment = .040875,
The square of the diam. = 10000,
The area required = 408.75.

PROBLEM XIV.

PAGE 80.

To find the area of a zone.

Ex. 2. Here $\frac{96-60}{2}=\frac{36}{2}=18$ = half the difference between the two chords,

2*

And $26+\frac{(96-18)\times 18}{26}=26+54=80=DF$. (See the figure in the work to this Problem.)

Also $\sqrt{80^2+60^2}=100=$the diameter of the circle.

Then $\frac{100-\sqrt{100^2-60^2}}{2}=\frac{100-80}{2}=10=$versed sine of half the less arc,

And $26+10=36=$versed sine of half the greater arc,

Also $\frac{10}{100}=.1=$tab. v. sine, the cor. seg. is .040875,

And $\frac{36}{100}=.36=$ " " " " .254550,

Difference .213675.

Which, multiplied by 100^2, gives 2136.75, the area of the zone. Ans.

Ex. 3. Here $\frac{48-30}{2}=\frac{18}{2}=9=$ half the difference between the two chords,

And $13+\frac{(48-9)\times 9}{13}=13+27=40=DF$.

Also $\sqrt{40^2+30^2}=\sqrt{2500}=50=$ the diameter of the circle.

Then $\frac{50-\sqrt{50^2-30^2}}{2}=\frac{50-40}{2}=5=$versed sine of less segment.

And $5+13=18=$versed sine of greater segment.

Also $\frac{5}{50}=.1$ tab. ver. sine, cor. segment, .040875,

And $\frac{18}{50}=.36$ " " " " .254550,

Difference .213675.

Which, multiplied by 50^2, gives 534.1875. Ans.

PROBLEM XV.

PAGE 82.

To find the area of a circular ring.

Ex. 2. Here $\overline{16+10} \times \overline{16-10} \times .7854 = 26 \times 6 \times .7854 =$ 122.5224. Ans.

Ex. 3. Here $\overline{21.75+9.5} \times \overline{21.75-9.5} \times .7854 = 31.25 \times$ $12.25 \times .7854 = 300.6609$. Ans.

Ex. 4. Here $\overline{6+4} \times \overline{6-4} \times .7854 = 10 \times 2 \times .7854 =$ 15.708. Ans.

PROBLEM XVI.

PAGE 83.

To find the area of a lune.

Ex. 2. Here the area of the lesser segment is 26.8783 (Prob. XIII., Rule 1., Ex. 2).

Also $\frac{10^2+10^2}{10} = \frac{200}{10} = 20 =$ diameter of greater segment,

And $\frac{10}{20} = .5 =$ tab. versed sine; cor. segment is .392699, which, multiplied by 20^2, gives 157.0796 for the area of the greater segment; hence the lune $= 157.0796 - 26.8783 =$ 130.2013. Ans.

Note.—Since the versed sine of the larger segment is equal to half the chord, it is evident that the segment is a semicircle; hence its area may be more concisely calculated by taking half the area of a circle whose diameter is equal to the chord.

Ex. 3. Here the diameter of the less segment is $\frac{24^2+7^2}{7}$

$=\frac{576+49}{7}=\frac{625}{7}$; and $7\div\frac{625}{7}=\frac{49}{625}=.0784$; the tabular segment corresponding to .078 is .028356; that corresponding to .079 is .028894; their difference 538 multiplied by .4 gives 215.2, which added to .028356, gives .028571, the segment corresponding to .0784.

Also $.028571\times\left(\frac{625}{7}\right)^2=.028571\times\frac{390625}{49}=\frac{11160.546875}{49}$ $=227.7663$ the area of the less segment.

Again, the diameter of the greater segment is $\frac{24^2+18^2}{18}$ $=\frac{900}{18}=50$, and $\frac{18}{50}=.36$ tab. versed sine, corresponding segment is .254550, which, multiplied by 50^2, gives 636.375, the area of the greater segment.

Hence $636.375-227.7663=408.6087$ the area required.

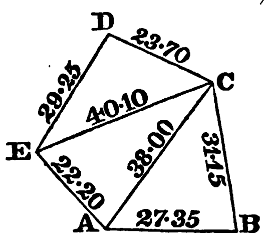
PROBLEM XVII.

PAGE 84.

To find the area of an irregular polygon.

Ex. 2. The annexed figure is a representation of this field, with the sides respectively marked in chains and links.

Note.—Links are brought to chains by dividing them by 100.



To find the area of the triangle ABC.

	ch.
AB=	27.35
BC=	31.15
AC=	38.00
sum=	96.50
half sum=	48.25
half sum—AB=	20.90
half sum—BC=	17.10
half sum—AC=	10.25

Hence $\sqrt{(48.25\times 20.90\times 17.10\times 10.25)}=\sqrt{1008.4250\times 175.2750}=\sqrt{176751.691875}=420.41847$ = area of the triangle ABC.

For area of ACE.

	ch.
AC=	38.00
CE=	40.10
EA=	22.20
sum=	100.30
half sum=	50.15
half sum—AC=	12.15
half sum—CE=	10.05
half sum—EA=	27.95

Hence $\sqrt{(50.15\times 12.15\times 10.05\times 27.95)}=\sqrt{609.3225\times 280.8975}=\sqrt{171157.16694375}=413.71145$ = area of the triangle ACE.

For the area of ECD:

	ch.
EC=	40.10
CD=	23.70
DE=	29.25
sum=	93.05
half sum=	46.525
half sum−EC=	6.425
half sum−CD=	22.825
half sum−DE=	17.275

Hence $\sqrt{(46.525 \times 6.425 \times 22.825 \times 17.275)} = \sqrt{298.923125 \times 394.301875} = \sqrt{117865.948668359375} = 343.3161$ = area of the triangle ECD.

	sq. ch.
Now area ABC=	420.41847
ACE=	413.71145
ECD=	343.3161

Their sum is the area of the whole tract ABCDEA $\Big\}$ = 1177.44602 = 117 *ac.* 2 *ro.* 39 *po.* Ans.

PROMISCUOUS QUESTIONS.

PAGE 86.

Ex. 3. Here $(25)^2 \times .7854 = 490.875$, the area of the circle whose diameter is 25, and $490.875 \times 9 = 4417.875$, the area of a circle which contains 9 times as much:

Hence we have $490.875 : 4417.875 :: 625 : 5625$.

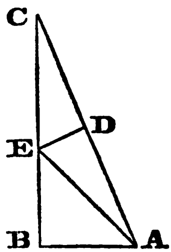
And $\sqrt{5625} = 75$, the diameter,

Therefore, $\frac{75}{2} = 37.5$, the length of the string.

Ex. 4. Here, if the bottom of the ladder be moved 10 feet from the wall, it is evident that the altitude of the top end will $= \sqrt{100^2 - 10^2} = \sqrt{9900} = 99.4987437$.

Hence $100 - 99.4987437 = .5012563 =$ the distance the top of the ladder will move down the wall.

Ex. 5. Here the square of the diameter $= 5028\frac{4}{7} \div .7854 = \frac{35200}{7} \div .7854 = \frac{35200}{7 \times .7854} = \frac{35200}{5.4978}$. But the square of half any line is equal to one-fourth the square of the whole line; therefore, the square of the radius $= \frac{1}{4}$ of $\frac{35200}{5.4978} = \frac{4400}{2.7489}$. Make AB, in the annexed figure, $= \sqrt{\frac{4400}{2.7489}}$, and draw BC at right angles to it, and equal to the height of the pole. Join AC and bisect it in D. Draw DE perpendicular to AC, and join EA. Then will E represent the place at which the pole was broken. For in the two tri-



angles ADE and CDE, CD=DA, and DE is common to both the triangles. Also the contained angle ADE=the angle CDE; wherefore (4.1.) CE=EA. Q. E. D.

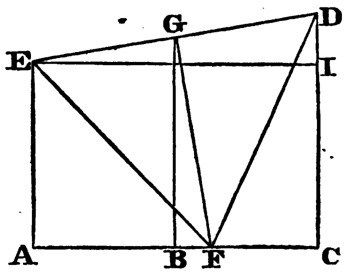
$$\text{Now } CA^2=AB^2+BC^2=\frac{4400}{2.7489}+100^2=\frac{31889}{2.7489}.$$

Also, by similar triangles, CB : CA : : $\frac{CA}{2}$ (CD) : CE

$$=\frac{CA^2}{2BC}=\frac{31889}{2.7489}\div 200=\frac{31889}{549.78}=58.0032.$$

Hence BE=BC—CE=100—58.0032 = 41.9968 = the height of the broken piece required.

Ex. 6. Let AE and CD, in the annexed figure, represent the lower and higher trees respectively. Join ED. and on the middle of it erect the perpendicular GF, then F will represent the place of the fountain. For join EF and DF, then the two sides EG, GF, are equal to the two DG, GF, and the angle EGF is equal to the angle DGF; wherefore (4.1.) EF=DF. Q. E. D.



Calculation.—Draw EI parallel to AC, and GB parallel to AE. Then $GB=\frac{DC+AE}{2}=90$, DI = DC—AE = 20, and AB=BC=½ AC=60.

Again (29.1.) $\angle$AED=$\angle$BGD, from which take the right angles AEI and FGD respectively, and the $\angle$IED =$\angle$BGF. Wherefore the two triangles EID and BGF

are similar; hence EI (120) : ID (20) : : GB (90) : BF = 15. Now AF = AB + BF = 75, and CF = CB − BF = 45 = the distances from the fountain to the two trees.

Lastly, DF $= \sqrt{CD^2 + CF^2} = \sqrt{100^2 + 45^2} = \sqrt{12025}$ $= 5\sqrt{481} = 109.6585$ = distance of the fountain from each ball.

Ex. 7. Here, first find the area of a triangle whose three sides are 9, 8, and 6. The half sum of the three sides is equal to $\frac{9+8+6}{2} = 11.5$; hence the differences will be 2.5, 3.5, and 5.5, respectively,

And $\sqrt{11.5 \times 2.5 \times 3.5 \times 5.5} = \sqrt{553.4375} = 23.5252$,

Also 6 *ac.* 1 *ro.* 12 *po.* = 1012 *per.* Wherefore 23.5252 : 1012 : : 8^2 : 2753.1332, and $\sqrt{2753.1332} = 52.47$ = the side in the large triangle which is homologous to 8 in the small one.

$$\left.\begin{array}{l} \text{Now } 8 : 9 : : 52.47 : 59.029 \\ \text{And } 8 : 6 : : 52.47 : 39.353 \end{array}\right\} \text{the two other sides.}$$

Ex. 8. Here 6 *ac.* 0 *ro.* 12 *per.* = 972 *per.* Hence, by note, page 54, the perpendicular height $= \frac{972 \times 2}{72} = 27$. Now, since the triangle is isosceles, the square of the side will be equal to the square of half the base added to the square of the perpendicular height, that is, the side $= \sqrt{36^2 + 27^2}$ $= \sqrt{2025} = 45$. Ans.

CONIC SECTIONS.

Of the Ellipse.

PROBLEM II.

CASE I.—PAGE 94.

Ex. 2. Here $35-28=7$, and $\sqrt{28\times 7}=\sqrt{196}=14$,
Then $35 : 25 :: 14 : 10$ = the ordinate required.

CASE II.—PAGE 95.

Ex. 2. Here $\sqrt{12.5^2-10^2}=\sqrt{56.25}=7.5$ = the square root of the difference of the squares of the ordinate and semi-conjugate,

And $25 : 35 :: 7.5 : 10.5$ = the distance between the ordinate and centre,

Hence semi-transverse $17.5+10.5=28$ = the greater abscissa,

And semi-transverse $17.5-10.5=7$ = the less abscissa.

CASE III.—PAGE 96.

Ex. 2. Here $12.5-\sqrt{12.5^2-10^2}=12.5-7.5=5$,
Then $5 : 7 :: 25 : 35$ = the transverse.

CASE IV.—PAGE 97.

Ex. 2. Here $35-7=28$ = the greater abscissa, and $\sqrt{28\times 7}=14$ = the square root of the product of the two abscissas.

Then $14 : 10 :: 35 : 25$ = the conjugate.

PROBLEM III.

PAGE 99.

To find the circumference of an ellipse.

Ex. 2. Here $\sqrt{\left(\frac{24^2+18^2}{2}\right)} \times 3.1416 = \sqrt{450} \times 3.1416 =$ $21.2132 \times 3.1416 = 66.6434$. Ans.

PROBLEM IV.

PAGE 99.

To find the area of an ellipse.

Ex. 2. Here $25 \times 35 \times .7854 = 875 \times .7854 = 687.2250 =$ area required.

Ex. 3. Here $70 \times 50 \times .7854 = 3500 \times .7854 = 2748.9 =$ area required.

PROBLEM V.

PAGE 100.

To find the area of an elliptic segment.

Ex. 2. Here $60 - 36 = 24 =$ the height of the segment,

And $\frac{24}{120} = .200 =$ tabular versed sine, the segment corresponding to which, from the table, is .111823,

Hence $.111823 \times 120 \times 40 = 4800 \times .111823 = 536.7504$ $=$ area required.

Ex. 3. Here $\frac{5}{25} = .200 =$ tabular versed sine, the segment corresponding to which, from the table, is .111823,

Hence $.111823 \times 35 \times 25 = .111823 \times 875 = 97.845125 =$ area required.

Of the parabola.

PROBLEM VII.

PAGE 103.

Ex. 2. First, as $9 : 6^2 : : 16 : 64$; and $\sqrt{64} = 8 =$ the other ordinate.

Second, as $16 : 8^2 : : 9 : 36$; and $\sqrt{36}=6=$the other ordinate.

Third, as $8^2 : 16 : : 6^2 : 9=$the other abscissa.

Fourth, as $6^2 : 9 : : 8^2 : 16=$the other abscissa.

PROBLEM IX.

PAGE 105.

To find the area of a parabola.

Ex. 2. Here $\frac{2}{3}\times(38\times 12)=\frac{2\times 456}{3}=\frac{912}{3}=304=$ area required.

Ex. 3. Here the base is equal to twice the ordinate$=16$. Hence $\frac{2}{3}\times\overline{16\times 10}=\frac{2\times 160}{3}=\frac{320}{3}=106\frac{2}{3}=$ area required.

PROBLEM X.

PAGE 106.

To find the area of the frustum of a parabola.

Ex. 2. Here $\frac{24^3-20^3}{24^2-20^2}=\frac{13824-8000}{576-400}=\frac{5824}{176}=\frac{364}{11}$.

And $\frac{2}{3}$ of $5\frac{1}{2}=\frac{2}{3}$ of $\frac{11}{2}=\frac{22}{6}=\frac{11}{3}$.

Hence $\frac{364}{11}\times\frac{11}{3}=\frac{364}{3}=121.3333$=area required.

Ex. 3. Here $\frac{10^3-6^3}{10^2-6^2}=\frac{1000-216}{100-36}=\frac{784}{64}=\frac{49}{4}$,

And $\frac{2}{3}$ of $4=\frac{8}{3}$.

Hence $\frac{49}{4}\times\frac{8}{3}=\frac{392}{12}=32\frac{2}{3}$=area required.

Of the Hyperbola.

PROBLEM XII.

CASE I.—PAGE 108.

Ex. 2. Here the transverse diameter $24+8=32$=the greater abscissa,

And $\sqrt{32\times8}=\sqrt{256}=16$=the square root of the product of the two abscissas.

Hence $24:21::16:14$=the ordinate required.

Ex. 3. Here $50+12=62$=the greater abscissa,

And $\sqrt{62\times12}=\sqrt{744}=27.2764$ = square root of the product of the two abscissas.

Hence $50:30::27.2764:16.3658$ = the ordinate required.

CASE II.—PAGE 109.

Ex. 2. Here $\sqrt{14^2+10.5^2}=\sqrt{306.25}=17.5$ = the square root of the squares of the ordinate and semi-conjugate,

And $21:24::17.5:20$ = the distance between the ordinate and centre.

Hence the greater abscissa=the semi-transverse$+20=12+20=32$,

And the less abscissa$=20-$the semi-transverse$=20-12=8$. Ans.

Ex. 3. Here $\sqrt{24^2+18^2}=\sqrt{900}=30=$the square root of the sum of the squares of the ordinate and semi-conjugate,

And $36 : 60 :: 30 : 50=$the distance between the ordinate and centre.

Hence $50 +$ semi-transverse $= 50 + 30 = 80 =$ greater abscissa,

And $50-$semi-transverse$=50-30=20=$less abscissa.

CASE III.—PAGE 111.

Ex. 2. Here $\sqrt{32\times 8}=\sqrt{256}=16=$the square root of the product of the two abscissas.

Hence $16 : 14 :: 24 : 21=$the conjugate required.

CASE IV.—PAGE 112.

Ex. 2. Here $\sqrt{14^2+10.5^2}=\sqrt{306.25}=17.5$; and $17.5+10.5=28$.

Then $14^2 : 21\times 8 :: 28 : 24=$the transverse required.

Ex. 3. Here $\sqrt{24^2+18^2}=\sqrt{900}=30$, and $30+18=48$.

Then $24^2 : 36\times 20 :: 48 : 60=$transverse required.

PROBLEM XIII.

PAGE 113.

To find the length of any arc of an hyperbola, beginning at the vertex.

Ex. 2. Here $120 : 72 :: 72 : 43.2=$the parameter

Then, First, $(\overline{120\times 19}+\overline{43.2\times 21})\times \frac{40}{120}=3187.2\times \frac{1}{3}=$ 1062.4.

Second, $(\overline{120\times 9}+\overline{43.2\times 21})\times \frac{40}{120}=1987.2\times \frac{1}{3}=662.4.$

Third, $\frac{1062.4+\overline{43.2\times 15}}{662.4+(43.2\times 15)}\times 48=\frac{1710.4}{1310.4}\times 48=\frac{82099.2}{1310.4}$ $=62.6520$. Ans.

Note.—The parameter is always a third proportional to the transverse and conjugate diameters.

Ex. 3, Here, by Problem XII., Case 2, find the less abscissa.

$\sqrt{10^2+30^2}=\sqrt{1000}=31.6228=$ the square root of the sum of the squares of the ordinate and semi-conjugate.

Now 60 : 80 : : 31.6228 : 42.1637 = the distance between the ordinate and centre.

Hence the less abscissa = 42.1637 — semi-transverse = $42.1637-40=2.1637$.

As 80 : 60 : : 60 : 45 = parameter.

Then, First, $(\overline{80\times 19}+\overline{45\times 21})\times \frac{2.1637}{80}=2465\times \frac{2.1637}{80}=\frac{5333.5205}{80}=66.6690.$

Second, $(\overline{80\times 9}+\overline{45\times 21})\times \frac{2.163}{80}=1665\times \frac{2.1637}{80}=\frac{3602.5605}{80}=45.0320.$

Third, whence $\frac{66.6690+\overline{45\times 15}}{45.0320+(45\times 15)}\times 10=\frac{741.6690}{720.0320}\times 10=\frac{741.6690}{720.0320}=10.3005=$ the length of the hyperbola reckoned from the vertex.

Hence $10.3005\times 2=20.6010=$ the whole length required.

Note.—The difference between the results to these two questions here obtained, and those given in the work, is caused by using vulgar instead of decimal fractions.

PROBLEM XIV.

PAGE 115.

To find the area of an hyperbola.

Ex. 2. Here $21\sqrt{100\times50+\frac{5}{7}\times50^2}=21\sqrt{5000+\frac{12500}{7}}$

$=21\sqrt{\frac{47500}{7}}=1729.8844.$ $\frac{1729.8844+4\sqrt{100\times50}}{75}=\frac{1729.8844+4\times70.7107}{75}$

Ans. $\frac{1729.8844+4\ldots}{75}$ ... $26.8\ldots2\times120=3220$

$=26.8\ldots$ $\frac{4}{10}$ ... $50\ldots60$

Then 2... $=21\sqrt{7250+\frac{8125}{7}}=$

... Ans.

... $35.8\ldots$

...

SOLIDS.

PROBLEM I.

PAGE 121.

To find the solidity of a cube.

Ex. 2. Here 15 *in.* $=1.25$ *ft.*
And $1.25^3=1.953125=1$ *ft.* 11 *in.* 5.25 *pa.* Ans.

Ex. 3. Here $\frac{17.5^3}{1728}=\frac{5359.375}{1728}=3.1015$ *ft.* Ans.

PROBLEM II.

PAGE 122.

To find the solidity of a parallelopipedon.

Ex. 2. Here 21 *in.* $=1.75$ *ft.*
Then $1.75\times1.75\times15=3.0625\times15=45.9375$ *ft.* Ans.

Ex. 3. Here $10\times5.75\times3.5=201.25$ *ft.*

PROBLEM III.

PAGE 123.

To find the solidity of a prism.

Ex. 2. Here, by Problem VIII. Superficies, the area of the triangular base is $=1.5^2\times.433013=2.25\times.433013=.97427925$,

And $.97427925 \times 18 = 17.5370265$ *ft.* Ans.

Ex. 3. Here, by Problem VIII. Superficies, the area of the base $= \overline{\tfrac{4}{3}}|^2 \times 2.598076 = \frac{16}{9} \times 2.598076 = \frac{41.569216}{9} = 4.618802$,

And $4.618802 \times 15 = 69.28203$ *ft.* Ans.

PROBLEM IV.

PAGE 124.

To find the convex surface of a cylinder.

Ex. 2. Here $3.1416 \times 30 = 94.248$ = the circumference of the base,

And $94.248 \times 60 = 5654.88$ *inches*=convex surface required.

Ex. 3. Here 8 *ft.* 4 *in.*$=8\tfrac{1}{3}$ *ft.*$=\tfrac{25}{3}$ *ft*

And $\tfrac{25}{3} \times 14 = \tfrac{350}{3} = 116.6666$, &c. *ft.* Ans.

PROBLEM V.

PAGE 125.

To find the solidity of a cylinder.

Ex. 2. Here $2^2 \times .7854 = 4 \times .7854 = 3.1416$ = area of the base,

And $3.1416 \times 5 = 15.708$ *ft.* Ans.

Ex. 3. Here $20^2 \times .07958 = 400 \times .07958 = 31.832$=area of the base,

And $31.832 \times 20 = 636.64$=solidity required.

Ex. 4. Here $20^2\times.07958=400\times.07958=31.832=$area of the base,

And $31.832\times19.318=614.930576$ *ft.* Ans.

PROBLEM VI.

PAGE 128.

To find the convex surface of a right cone.

Ex. 2. Here $4.5\times3.1416=14.1372=$the circumference of the base,

And $\frac{14.1372\times20}{2}=141.372=$convex surface required.

Ex. 3. Here $\frac{10.75\times18.25}{2}=\frac{196.1875}{2}=98.09375$. Ans.

PROBLEM VII.

PAGE 130.

To find the convex surface of the frustum of a right cone.

Ex. 1. Here $\frac{\overline{30+10}\times20}{2}=\frac{40\times20}{2}=400$ *ft.* Ans.

Ex. 2. Here $3.1416\times8=25.1328=$ the circumference of the greater end,

And $3.1416\times4=12.5664=$ the circumference of the less end.

Then their sum, $37.6992\times\frac{20}{2}=376.992$ *feet.* Ans.

Ex. 3. Here we have, by similar triangles, (see the figure to the Problem) CB (30) : BA (10) : : CD (6) : DE (2) $=$the circumference of the less end. Also BD$=$BC$-$DC $=30-6=24=$the slant height of the frustum.

Hence $\frac{\overline{10+2}\times24}{2}=144$ *ft.* surface required.

PROBLEM XIX.

PAGE 155.

To find the solidity of a parabolic conoid.

Ex. 2. Here $100^2 \times .7854 = 7854 =$ the area of the base,

And $7854 \times \frac{60}{2} = 7854 \times 30 = 235620 =$ solidity required.

Ex. 3. Here $40^2 \times .7854 = 1600 \times .7854 = 1256.64 =$ the area of the base;

And $1256.64 \times \frac{30}{2} = \frac{37699.2}{2} = 18849.6 =$ solidity required.

Ex. 4. Here $100^2 \times .7854 = 7854 =$ the area of the base,

And $7854 \times \frac{50}{2} = \frac{392700}{2} = 196350$. Ans.

PROBLEM XX.

PAGE 156.

To find the solidity of the frustum of a paraboloid.

Ex. 2. Here $(60^2 + 48^2) \times 18 \times .3927 = 5904 \times 18 \times .3927 = 106272 \times .3927 = 41733.0144 =$ solidity required.

PROBLEM XXI.

PAGE 157.

To find the solidity of an hyperboloid.

Ex. 2. Here $(52^2 + 68^2) \times 50 \times .5236 = 366400 \times .5236 = 191847.04$. Ans.

PROBLEM XXII.

PAGE 159.

To find the solidity of the frustum of an hyperbolic conoid.

Ex. 2. Here $(5^2+3^2+\overline{8.5}|^2)\times 12\times .5236=1275\times .5236=667.59$. Ans.

Ex. 3. Here $(8^2+6^2+\overline{14.5}|^2)\times 20\times .5236=6205\times .5236=3248.938$. Ans.

Ex. 4. Here $(4^2+6^2)\times 10\times .5236=520\times .5236=272.272$. Ans.

MISCELLANEOUS QUESTIONS.

PAGE 161.

Ex. 5. Here, since similar solids are as the cubes of their like dimensions, we shall have $\overline{4.5}|^3 : 1^3 :: 13.5 :$ $\frac{13.5}{\overline{4.5}|^3} = \frac{3}{\overline{4.5}|^2} = \frac{3}{20.25} = 3 \div 20\frac{1}{4} = 3 \div \frac{81}{4} = \frac{12}{81} = \frac{4}{27}$ = the content of the part cut off.

Hence $13\frac{1}{2} - \frac{4}{27} = \frac{27}{2} - \frac{4}{27} = \frac{729-8}{54} = \frac{721}{54}$ = the content of the hopper in cubic feet,

And $\frac{721}{54} \times 1728 = \frac{1245888}{54} = 23072$ = the content in cubic inches.

Now 2150.4 cubic inches make one bushel dry measure.

Wherefore $23072 \div 2150.4 = 10.7292$ = the content in bushels, which was required.

Ex. 6. Here $\frac{22+20}{2} = 21$ = the mean breadth of the ditch.

Then, by Problem II. of Solids, $1000 \times 9 \times 21 = 189000$ = capacity in cubic feet,

And $189000 \times 1728 = 326592000$ = capacity in cubic inches.

Hence $326592000 \div 282 = 1158127\frac{31}{47}$ = capacity in gallons. Ans.

Ex. 7. Here, First, by Problem IX. of Solids, the solidity

of the small frustum$=(5^2+3^2+5\times 3)\times\frac{12}{3}\times .7854=196$ $\times .7854=153.9384$ *inches.*

Then, by Observation to Example 4th, page 163, we have 153.9384 : 3666 : : 5^2 : 595.36801733=the square of the greater required diameter.

Hence $\sqrt{595.36801733}=24.4002$=the greater diameter.

Consequently 5 : 3 : : 24.4002 : 14.6401 = the smaller diameter.

REGULAR BODIES.

PROBLEM I.

PAGE 166.

To find the solidity of a tetraedron.

Ex. 2. Here $\frac{6^3}{12} \times \sqrt{2} = 18 \times 1.414 = 25.452 =$ solidity required.

PROBLEM II.

PAGE 167.

To find the solidity of an octaedron.

Ex. 3 Here $\frac{8^3}{3} \times \sqrt{2} = \frac{512}{3} \times 1.4142 = \frac{724.0704}{3} =$ 241.3568. Ans.

PROBLEM III.

PAGE 168.

To find the solidity of a dodecaedron.

Ex. 2. Here $\sqrt{\frac{21\sqrt{5}+47}{40}} = \sqrt{\frac{21 \times 2.2361 + 47}{40}} = \sqrt{\frac{93.9581}{40}} = \sqrt{2.3489525} = 1.53262,$

And $1.53262 \times 1^3 \times 5 = 1.53262 \times 5 = 7.6631$ = solidity required.

PROBLEM IV.

PAGE 171.

To find the solidity of an icosaedron.

Ex. 2. Here $\sqrt{\frac{3\sqrt{5}+7}{2}} = \sqrt{\frac{3 \times 2.2361 + 7}{2}} = \sqrt{\frac{13.7083}{2}}$ $= \sqrt{6.85415} = 2.61804$,

And $2.61804 \times 1^3 \times \frac{5}{6} = 2.61804 \times \frac{5}{6} = \frac{13.0902}{6} =$ 2.1817. Ans.

CYLINDRIC RINGS.

PROBLEM I.

PAGE 174.

To find the convex surface of a cylindric ring.

Ex. 2. Here $(4 + 18) \times 4 \times 9.8696 = 88 \times 9.8696 =$ 868.5248. Ans.

Ex. 3. Here $\overline{2+18} \times 2 \times 9.8696 = 40 \times 9.8696 = 394.784$ square inches. Ans.

PROBLEM II.

PAGE 175.

To find the solidity of a cylindric ring.

Ex. 2. Here $\overline{4+18} \times 2^2 \times 9.8696 = 88 \times 9.8696 = 868.5248$ = solidity required.

Ex. 3. Here $\overline{2 + 12} \times 1^2 \times 9.8696 = 14 \times 9.8696 =$ 138.1744. Ans.

Ex. 4. Here $\overline{4 + 16} \times 2^2 \times 9.8696 = 80 \times 9.8696 =$ 789.5680. Ans.

BRICKLAYERS' WORK.

PAGE 180.

Ex. 2. Here $62\frac{1}{2}$ *ft.* $\times$ 14 *ft.* 8 *in.* $=\frac{125}{2}\times 14\frac{2}{3}=\frac{125}{2}\times\frac{44}{3}=\frac{5500}{6}$,

And $\frac{5500}{6}\times\frac{5}{3}=\frac{27500}{18}=\frac{13750}{9}=1527\frac{7}{9}$.

Hence $1527\frac{7}{9}\div 272=5$ *ro.* $167\frac{7}{9}$ *ft.* $=5$ *ro.* 167 *ft.* 9 *in.* 4 *pa.* = the content required.

Ex. 3. Here $\overline{45+15}\times 2=60\times 2=120$ = the whole perimeter of the building.

Now $120\times 20=2400$ = the area of the wall as high as the gable,

And $15\times 6=90$ = the area of the two gables.

Hence $\overline{2400+90}=2490$ = the whole superficial content,

And $\left(2490\times\frac{4}{3}\right)\div 272=3320\div 272=12.2059$ *rods.*

Ans.

Note.—In this question the gables are only considered within the walls, but it is customary to reckon them the width of the building on the outside.

MASONS' WORK.

PAGE 181.

Ex. 2. Here $53.5 \times 12.25 \times 2 = 53.5 \times 24.5 = 1310.75$ *ft.* $= 1310$ *ft.* 9 *in.*, the content required.

Ex. 3. Here 5 *ft.* 7 *in.* $= 5\frac{7}{12}$ *ft.* $= \frac{67}{12}$, and 1 *ft.* 10 *in.* $= 1\frac{5}{6}$ *ft.* $= \frac{11}{6}$ *ft.*
Hence $\frac{67}{12} \times \frac{11}{6} = \frac{737}{72} =$ content of the slab in feet,
And $\frac{737}{72} \times 80$ *cts.* $= \frac{58960}{72}$ *cts.* $= \frac{7370}{9}$ *cts.* $= 8$ *dols.* 18 *cts.* $8\frac{8}{9}$ *m.* Ans.

Ex. 4. Here $60.75 \times 10.25 \times 2.5 = 622.6875 \times 2.5 = 1556.71875$ *feet.* Ans.

Ex. 5. Here

ft.	*in.*
4	6
3	2
0	9
13	6
14	3

And

	ft.	*in.*
	4	4
	1	9
	3	3
	4	4
Content of the two jambs=	7	7
Content of the mantel and slab=	14	3

Content of the whole chimney-piece $= 21$ 10 Ans.

Ex. 6. Here $\overline{58+26} \times 2 = 168 =$ the whole girt of the building,
And $168 \times 22 = 3696 =$ the surface of the wall to the gable.

Also $26 \times 12 = 312 =$ the surface of the two gables.

Hence $3696 + 312 = 4008$, and $\frac{4008}{16.5} = 242.91$ *perches* of mason work.

Hence 242.91×56 *cts.* $= \$136.03 =$ amount of the mason work, reckoning the walls 18 *inches* thick, and making no deductions for windows, doors, &c.

To find the quantity of stone in the wall.

15 *in.* $= 1.25$ *ft.*, and $26 - \overline{1.25 \times 2} = 26 - 2.5 = 23.5 =$ the width of the building within the walls.

Hence $\overline{58 + 23.5} \times 2 = 163 =$ whole perimeter.

$163 \times 22 = 3586 =$ content of the wall to the gable.

$26 \times 12 = 312 =$ content of the two gables,

And $3586 + 312 = 3898 =$ superficial content of the building, including doors and windows.

Now $4\frac{1}{2} \times 8 \times 2 = 72 =$ deduction to be made for the doors,

And $3\frac{1}{2} \times 6 \times 28 = 588 =$ deduction to be made for the windows.

Wherefore $3898 - (588 + 72) = 3898 - 660 = 3238$, and $3238 \times 1.25 = 4047.5 =$ content of the wall in solid feet; which, being divided by 24.75, will $= 163.535$ *perches*, the quantity of stone.

Hence 163.535×44 *cts.* $= \$71.95.5$, the cost of the stone.

Note.—Dividing by 16.5 *feet* brings it to perches when the wall is 18 *inches* thick, and by 24.75 *feet* when it is one *foot* thick.

CARPENTERS' WORK.

PAGE 185.

Ex. 2. Here $53.5 \times 47.75 = 2554.625\,ft. = 2554\,ft.\ 7\tfrac{1}{2}\,in. = 25\,sq.\ 54\,ft.\ 7\tfrac{1}{2}\,in.$ Ans.

Ex. 3. Here $91.75 \times 11.25 = 1032.1875\,ft. = 10\,sq.\ 32\,ft.$ Ans.

Ex. 4. Here $18.25 + \frac{18.25}{2} = 18.25 + 9.125 = 27.375 =$ the measure of the width of the roof, allowing for the true pitch,

And $27.375 \times 44.5 = 1218.1875\,ft. = 12\,sq.\ 18\,ft.$ Ans.

Ex. 5. Here $30.5 + \frac{30.5}{2} = 30.5 + 15.25 = 45.75 =$ the girt of the roof,

And $52\,ft.\ 8\,in. = 52\tfrac{2}{3}\,ft. = \frac{158}{3}$.

Also $45.75 \times \frac{158}{3} = \frac{7228.5}{3} = 2409.5\,ft. = 24.095$ *squares of roofing.*

Hence $24.095 \times 140\,cts. = 3373.3 = \$33.73.3$ Ans.

SLATERS' AND TILERS' WORK.

PAGE 187.

Ex. 2. Here 27 *ft.* 5 *in.* $= 27\frac{5}{12} = \frac{329}{12}$, and $\frac{329}{12} + \left(\frac{329}{12} \div 2\right) = \frac{329}{12} + \frac{164.5}{12} = \frac{493.5}{12} =$ the true pitch.

Also $16 \times 2 = 32$ *in.* $= \frac{32}{12}$ *ft.* to be added for the projection of the eave boards.

Hence $\frac{493.5}{12} + \frac{32}{12} = \frac{525.5}{12} =$ the girt of the roof.

Now 43 *ft.* 10 *in.* $= 43\frac{5}{6}$ *ft.* $= \frac{263}{6}$, and $\frac{525.5}{12} \times \frac{263}{6} = \frac{138206.5}{72}$ *ft.* $= \frac{1382.065}{72}$ *squares of roofing.*

Hence $\frac{1382.065}{72} \times 340$ *cts.* $= \frac{469902.1}{72}$ *cts.* $= \$65.26.4 =$ cost of roofing. Ans.

PLASTERERS' WORK.

PAGE 189.

Ex. 2. Here $141.5 \times 11.25 = 1591.875$ *sq. ft.*
And $1591.875 \div 9 = 176.875$ *square yards*. Ans.

Ex. 3. Here $\overline{14\,ft.\ 5\,in. + 13\,ft.\ 2\,in.} \times 2 = 27\,ft.\ 7\,in. \times 2 = 55\,ft.\ 2\,in.$ = the whole girt of the room.

	ft.	*in.*	
	55	2	
	9	3	
	13	9	6
	496	6	0
	510	3	6
Door=7×4=	28		
	9)482	3	6=content in feet.
Rendering=	53 *yds.* 5 *ft.* 3 *in.* 6 *pa.*		

Also $14\,ft.\ 5\,in. - 5\,in. \times 2 = 14\,ft.\ 5\,in. - 10\,in. = 13\,ft.\ 7\,in.$ = length of the ceiling,

And $13\,ft.\ 2\,in. - 5\,in. \times 2 = 13\,ft.\ 2\,in. - 10\,in. = 12\,ft.\ 4\,in.$ = breadth of the ceiling,

And $12\,ft.\ 4\,in. \times 13\,ft.\ 7\,in. = 167\,ft.\ 6\,in.\ 4\,pa. = 18$ *yds.* 5 *ft.* 6 *in.* 4 *pa.* of ceiling.

	ft.	*in.*		
Lastly,	55	2		=the length of the cornice,
		$8\frac{1}{2}$		=its girt.
	36	9	4	
	2	3	7	
	39	0	11	=39 *ft.* $0\frac{11}{12}$ *in.* of cornice

PAINTERS' WORK.

PAGE 191.

	ft.	*in.*		
Ex. 2. Here	14	10		
	21	8		
	9	10	8	
	311	6		
	321	4	8	=321.3888 *ft.*=35.71 *yds.* nearly. Ans.

	ft.	*in.*		
Ex. 3 Here	74	10		=room's girt.
	11	7		=height.
	43	7	10	
	823	2		
	866	9	10	=room.

ft.	*in.*		
7	6		
3	9		
5	7	6	
22	6		
28	1	6	=door.

ft.	in.	
6	8	
3	4	
2	2	8
20		
22	2	8
		5
111	1	4=shutters.

ft.	in.
6	9
5	0
33	9=chimney.

ft.	in.	
3	6=closet's depth.	
4	9=breadth.	
8	3	
	2	
16	6=inside compass.	
11	7=height.	
9	7	6
181	6	
191	1	6=inside area.

ft.	in.
22	6
	10
18	9
	2
37	6=shelves.

ft.	in.	
8	0	=height of the breaks.
3	4	=width.
11	4	
	2	
22	8	=girt of each break.
	14	=depth.
26	5	4=area of each break.
		5
132	2	8=area of all the breaks.

	ft.	in.	pa.
Room=	866	9	10
Door=	28	1	6
Shutters=	111	1	4
Breaks=	132	2	8
Closet=	191	1	6
Shelving=	37	6	0
	1366	10	10
Chimney (*deduct*)	33	9	

Surface to be painted=1333 1 10=1333.153 *ft.*

And 1333.153÷9=148.128 *yds.*

8 *d.* price per *yd.*

12)1185.024

2|0)9|8 9.024

£4 18 9.024 Ans.

Note.—It is evident that one side of the door and shutters was included in the area of the room.

PAVIOURS' WORK.

PAGE 193.

Ex. 2. Here $42.75 \times 68.5 = 2928.375$ = the area of the whole yard in feet,

And $68.5 \times 5.5 = 376.75$ = the area of the footway in feet.

Their difference $= 2551.625$ = part to be laid with pebbles.

Hence $\frac{376.75}{9} \times 3.5\ s. = \frac{1318.625}{9} = 146.514\ s.$ = cost of paving the walk,

And $\frac{2551.625}{9} \times 3\ s. = 850.542\ s.$ = cost of pebbling the yard.

Their sum $= 997.056\ s = 49£\ 17s.\ 0\frac{1}{2}d.$ Ans.

VAULTED AND ARCHED ROOFS.

PROBLEM I.

PAGE 195.

To find the solid content of circular, elliptic, or gothic vaulted roofs.

Ex. 2. Here $12 \times 40 \times .7854 = 376.992$ = area of one end,
And $376.992 \times 80 = 30159.36$ = solidity required.

Ex. 3. The area of the segment of a circle whose chord is 48, and height 18, is found (by Rule 2, Prob. XIII.) to be 636.375; hence, the area of both segments is equal to $636.375 \times 2 = 1272.75$.

And the area of the triangle, $.433013 \times 48^2 = 997.661952$
Area of the segments $= 1272.75$

Multiply 2270.411952
by 60

136224.717120

= the solidity required.

PROBLEM III.

PAGE 196.

To find the solid content of a dome.

Ex. 2. The area of a hexagon whose side is $1 = \frac{3}{2}\sqrt{3}$.
Hence $\frac{3}{2}\sqrt{3} \times 20^2 = 600\sqrt{3}$ = the area of the base.

Also $\sqrt{20^2 - 10^2} = \sqrt{300} = 10\sqrt{3} =$ perpendicular height.

And $\frac{2}{3}$ of $10\sqrt{3} = \frac{20\sqrt{3}}{3} = \frac{2}{3}$ of the perpendicular height.

Wherefore $600\sqrt{3} \times \frac{20\sqrt{3}}{3} = \frac{12000 \times 3}{3} = 12000$. Ans.

* *Note.*—The area of the base may be found by Problem VIII. Superficies.

PROBLEM V.

PAGE 200.

To find the solid content of a saloon.

Ex. 2. Here the height of the arch is 5 *feet*, and its projection 5 *feet*, the diameter of the ceiling$=40-5\times 2=30$, and its perimeter$=30\times 3.1416=94.248$.

Hence $5 \times 5 \times \frac{94.248}{4} \times 3,1416 = 589.05 \times 3.1416 = 1850.55948 = A$,

And $\frac{2}{3} \times 5 \times \left((40-30)^2 \times .7854\right) = \frac{10}{3} \times 78.54 = \frac{785.4}{3} = 261.8 = B$,

$30^2 \times .7854 \times 5 = 706.86 \times 5 = 3534.3 =$ cylindrical part.

Also $40^2 \times .7854 \times 20 = 1256.64 \times 20 = 25132.8 =$ content of the lower part of the room.

Hence the sum of these four quantities$=30779.45948$ *feet*$=$the capacity of the room required.

PROBLEM VI.

PAGE 200.

To find the solid content of the vacuity formed by a groin arch.

Ex. 2. Here $20^2 = 400 =$ the area of the base,

And $400 \times 6 \times .904 = 2400 \times .904 = 2169.6$. Ans.

PROBLEM VII.

PAGE 202.

To find the concave superficies of a circular groin.

Ex. 2. Here $9^2=81=$the area of the base,
And $81\times1.1416=92.4696=$superficies required.

TIMBER MEASURE.

PROBLEM I.

PAGE 207.

To find the area of a board or plank

	ft.	in.	
Ex. 2. Here	1	10	
	5	7	
	1	0	10
	9	2	
	10	2	10=content required.

Ex. 3. Here 12 *ft.* 6 *in.* $\times$ 11 *in.* = 11 *ft.* 5 *in.* 6 *pa.* = 11 *ft.* 5.5 *in.* = $11.458\frac{1}{3}$ *ft.*

And $11.458\frac{1}{3} \times 1.5d. = 17.1875d.$ = 1*s.* 5*d.* Ans.

Ex. 4. Here $\frac{18+11\frac{1}{4}}{2} = \frac{29.25}{2} = 14.625$ = the breadth of each of the last two in the middle,

And $13\frac{1}{2} \times 2 + 14\frac{1}{2} + 14.625 \times 2 = 27 + 14.5 + 29.25 =$ 70.75 *in.* = sum of the mean breadths.

Hence $\frac{70.75 \times 17.5}{12} = \frac{1238.125}{12}$ = content in feet,

And $\frac{1238.125}{12} \times 3 = \frac{1238.125}{4} = 309.53125\,d.$ = 1£. 5*s.* $9\frac{1}{2}$*d.* Ans.

PROBLEM VII.

PAGE 202.

To find the concave superficies of a circular groin.

Ex. 2. Here $9^2=81=$the area of the base,
And $81\times1.1416=92.4696=$superficies required.

TIMBER MEASURE.

PROBLEM I.

PAGE 207.

To find the area of a board or plank

Ex. 2. Here

ft.	*in.*	
1	10	
5	7	
1	0	10
9	2	
10	2	10=content required.

Ex. 3. Here 12 *ft.* 6 *in.* × 11 *in.*=11 *ft.* 5 *in.* 6 *pa.*=11*ft.* 5.5 *in.*=11.458⅓ *ft.*

And 11.458⅓ × 1.5*d.*=17.1875*d.*=1*s.* 5*d.* Ans.

Ex. 4. H... $\frac{18+11\frac{1}{4}}{2}=\frac{29.25}{2}$=14.625=the breadth of

...ch of ... o in the middle,

And ... 14½+14.625×2 = 27+14.5+29.25 =

...75 ... the mean breadths.

... 17.5 = $\frac{1238.125}{12}$=content in feet,

...5 × ... 25 =309.53125 *d.* = 1£. 5*s.*

Ex. 4. Here $\frac{9.43+7.92+6.15+4.74+3.16}{5}=\frac{31.40}{5}=$ 6.28=mean girt,

And $\left(\frac{6.28}{5}\right)^2=\frac{39.4384}{25}=1.577536=$square of $\frac{1}{4}$ of the girt,

Hence $1.577536 \times 17\frac{1}{4} \times 2 = 1.577536 \times 34.5 = 54.424992$. Ans.

SPECIFIC GRAVITY.

PROBLEM I.

PAGE 217.

To find the magnitude of a body from its weight being given.

Ex. 2. Here, As 922 *oz.* : 16 *oz.* : : 1728 *in.* : 30 *in.* nearly. Ans.

Ex. 3. 1 *ton*=35840 *oz.*
And 925 *oz.* : 35840 *oz.* : : 1 *ft.* : $38\frac{138}{185}$ *cubic feet.* Ans.

PROBLEM II.

PAGE 218.

To find the weight of a body from its magnitude being given.

Ex. 2. Here 1728 *in.* : 35.25 *in.* : : 922 *oz.* : 19 *oz.* nearly. Ans.

Ex. 3. Here $10\times3\times2.5=75$ *ft.* = the solidity of the block.
Hence 1 *ft.* : 75 *ft.* : : 925 *oz.* : 69375 *oz.*=$4335\frac{15}{16}$ *lbs.* Ans.

MISCELLANEOUS QUESTIONS.

PAGE 226.

Ex. 1. Here $48\times 30=1440=$ the content of the largest floor,

And $\overline{24\times 15}\times 2=720=$ the content of two others, each of half the dimensions.

Their difference $=720$ *ft.* or half the first content.

Ex. 2. Here $1\frac{1}{2}$ *yds.* $=13\frac{1}{2}$ *sq. ft.* $=\frac{27}{2}$ *sq. ft.*

And 26 *in.* $=2$ *ft.* 2 *in.* $=2\frac{1}{6}$ *ft.* $=\frac{13}{6}$ *ft.*

Hence $\frac{27}{2}\div\frac{13}{6}=\frac{162}{26}=\frac{81}{13}=6.23$ *ft.* Ans.

Ex. 3. Here $8\frac{1}{2}\times 3\frac{1}{2}=\frac{17}{2}\times\frac{7}{2}=\frac{119}{4}=29.75$ *sq. in.* $=$ area of an end of the joist,

And $29.75\times 2=59.50=$ area of an end of the scantling.

Hence $\frac{59.50}{4.75}=12.52$ *in.* deep. Ans.

Ex. 4. Here 24 *ft.* 8 *in.* $=24\frac{2}{3}$ *ft.* $=\frac{74}{3}$ *ft.*, and 14 *ft.* 6 *in.* $=14\frac{1}{2}$ *ft.* $=\frac{29}{2}$ *ft.*

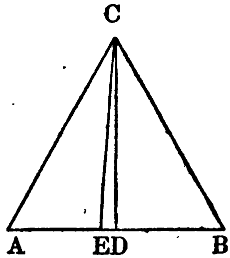
Hence $\frac{74}{3}\times\frac{29}{2}=\frac{2146}{6}=$ the content of the roof in square feet,

And $\frac{2146}{6}\times 8=\frac{17168}{6}=\frac{8584}{3}=$ the number of *lbs.* of lead that would be required to cover it.

Also $\frac{8584}{3}\div 112=\frac{8584}{336}=\frac{1073}{42}=$ quantity of lead in *cwts.*

Wherefore $\frac{1073}{42}\times 18$ *s.* $=\frac{19314}{42}=\frac{3219}{7}=459\frac{6}{7}$ *s.* $=22$£. 19 *s.* $10\frac{1}{4}$ *d.* Ans.

Ex. 5. Here, since the triangle is to be equilateral, it is evident that there will be the same number of feet in each side that there are yards in the whole perimeter. Hence the palisading will amount to a guinea for each foot in one side.



Let ABC represent the triangle, and let fall the perpendicular CD. Take DE=1 *ft.*, and join CE. Now it is plain that there will be just as many triangles equal to CED in the whole triangle, as there are feet in the side AB; and since the palisading amounts to a guinea for each foot in the side AB, the paving of each of these triangles must be a guinea.

Hence 1 *guinea* divided by 8*d.* $= \frac{252}{8} = 31.5 =$ the area of each of these triangles in feet whose base is 1 *ft.*

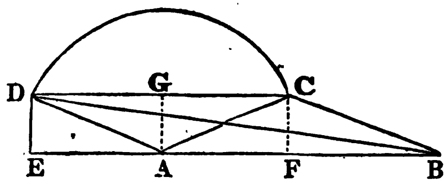
And $\frac{31.5 \times 2}{1} = 63$ *ft.* = the perpendicular height of the triangle ABC or EDC. Now the perpendicular of an equilateral triangle, whose side is 1, is easily found to be $= \frac{1}{2}\sqrt{3}$; Hence $\frac{1}{2}\sqrt{3} : 1 :: 63 : \frac{2 \times 63}{\sqrt{3}} = \frac{126}{\sqrt{3}} = 42\sqrt{3} =$ 72.746. Ans.

Or more concisely thus:

From Table, page 63, take the area of an equilateral triangle whose side is 1 *ft.*=.433013. Now the perimeter of this triangle is 3 *ft.* or 1 *yd.*, and consequently the cost of palisading it would be 1 *guinea*, or 252*d.*

But the cost of paving it$=.433013\times 8=3.464104\,d.$
Hence, As $3.464104\,d. : 252\,d. :: 1\,ft. : 72.746\,ft.$ Ans.

Ex. 6. Here it will be proper to construct the figure:



Make the base AB$=40$, and erect the perpendicular AG equal to twice the area divided by the base$=8$. Through G draw a line parallel to AB. With the centre A and radius 20, describe an arc cutting the parallel line in D and C. Join DA, DB, and CA, CB, then will ADB or ACB be the required triangle (37 and 41.1).

Produce BA, and let fall the perpendiculars DE and CF, then it is evident that $EA=AF=\sqrt{AC^2-CF^2}=\sqrt{20^2-8^2}=\sqrt{336}=18.3303.$

Also $BF=BA-AF=40-18.3303=21.6697$,

And $BE=BA+EA=40+18.3303=58.3303$.

Again, $BC=\sqrt{BF^2+FC^2}=\sqrt{21.6697^2+8^2}=\sqrt{533.57589809}=23.0993$,

And $BD=\sqrt{BE^2+ED^2}=\sqrt{58.3303^2+8^2}=\sqrt{3466.42389809}=58.8763$. Ans.

Ex. 7. Here the perpendicular, by making the longest side the base, is $=\frac{120\sqrt{3}}{20}=6\sqrt{3}$; hence the figure may be constructed as in the preceding example; it will, however, be found that the triangle ABC in this case will be acute, wherefore it will admit of but one form.

Now as in the last example, $AE=\sqrt{AD^2-ED^2}=\sqrt{12^2-(6\sqrt{3})^2}=\sqrt{144-108}=\sqrt{36}=6$,

And $BE=BA+AE=20+6=26$.

Also $BD=\sqrt{BE^2+ED^2}=\sqrt{26^2+(6\sqrt{3})^2}=\sqrt{676+108}=\sqrt{784}=28$. Ans.

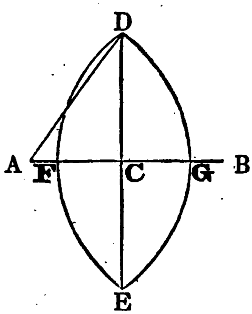
Ex. 8. Here, by Problem III. page 55, the area of the triangle is found to be 84; hence the areas of the triangles are as 84 : 24, or as 14 : 4; wherefore, as in Example 2, page 87, and the note at the bottom of page 162, we have

$$\sqrt{14}:\sqrt{4}::13:{}^*\frac{2\times 13}{\sqrt{14}}=\frac{26\sqrt{14}}{14}=\frac{13}{7}\sqrt{14};$$

$$\text{And } \sqrt{14}:\sqrt{4}::14:{}^*\frac{2\times 14}{\sqrt{14}}=\frac{28\sqrt{14}}{14}=2\sqrt{14}.$$

$$\text{Also } \sqrt{14}:\sqrt{4}::15:{}^*\frac{2\times 15}{\sqrt{14}}=\frac{30\sqrt{14}}{14}=\frac{15}{7}\sqrt{14}.$$

Ex. 9. Here $CF=CG=AG-AC=$ radius less half the distance of the centres $=25-15=10=$ the height of each segment.



* This fraction is reduced to the next following by multiplying the numerator and denominator by $\sqrt{14}$.

Now, by Rule II, in Problem XIII. of Superficies, the area of one of the segments is found thus: $\frac{10}{50}=.200=$ tab. versed sine, the segment corresponding to which is .111823, and this multiplied by $50^2=279.5575$, the area of one segment. Hence $279.5575\times 2=559.1150=$ the area of the space DFEG required.

Ex. 10. Here, by note, page 74, the radius of the circle $=\frac{30^2+10^2}{2\times 10}=50.$

Now the chord DE (see figure to the last question) $=$ 90 and $DC=\frac{90}{2}=45.$

Hence $AC=\sqrt{AD^2-DC^2}=\sqrt{50^2-45^2}=\sqrt{475}=$ 21.7945, and $CG=AG-AC=50-21.7945=28.2055.$

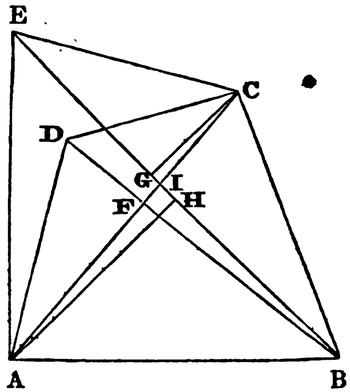
Ans.

Ex. 11. Here the area of an equilateral triangle whose side is $1=\frac{1}{4}\sqrt{3}$ or $\frac{\sqrt{3}}{4}$, and the perpendicular height $=\frac{1}{2}$ $\sqrt{3}$ or $\frac{\sqrt{3}}{2}$, the square of which is $\frac{3}{4}$.

Hence, as in Example 2, page 87, $\frac{\sqrt{3}}{4}:100::\frac{3}{4}:\frac{300}{\sqrt{3}}$ $=\frac{300\sqrt{3}}{3}=100\sqrt{3}=$ the square of the radius of the circle.

Wherefore $2\sqrt{100\sqrt{3}}=20\sqrt[4]{3}=26.32148$, the diameter required.

Ex. 12. Here the sums of the squares of the opposite sides being equal, the figure may be constructed, and the question solved thus:



Construction.—Draw AB and AE at right angles, and each equal to one of the given sides (35); join BE, and from the points E and B, with radii equal to the two remaining opposite sides (25 and 31) respectively, describe arcs intersecting on the farther side of BE, from A, in C; join AC, and draw BF at right angles to it. With the centre C, and radius equal to the remaining side (19), describe an arc cutting BF produced, in D. Join AD and CD, then will ABCD be the figure required.

Demonstration.—By the question $AB^2+CD^2=BC^2+CE^2$, and since BD and AC cross each other at right angles, (47.1,) $AB^2+CD^2=BC^2+AD^2$; wherefore $AD^2=EC^2$, or $AD=EC$.

Hence in the two triangles ABD and EAC, we have the two sides BA, AD equal the two AE, EC, each to each, and the angles ABD and EAC equal (each being the complement of BAF), and EC and AD, similarly situated; wherefore BD=AC. Q. E. D.

Calculation.—On BE let fall the perpendiculars CG and AH; now $BE^2=AB^2+AE^2=35^2\times 2$; $BE=\sqrt{35^2\times 2}=35\sqrt{2}=49.4975$; and $AH=BH=\frac{1}{2}BE=$

24.7487; again (13.2) $BG = \frac{BC^2 + BE^2 - CE^2}{2BE} = \frac{31^2+2\times35^2-25^2}{2\times49.4975} = \frac{2786}{98.995} = 28.1428$; $GH = BG - BH = 28.1428 - 24.7487 = 3.3941$; $CG = \sqrt{BC^2 - BG^2} = \sqrt{31^2 - 28.1428^2} = \sqrt{168.98280816} = 12.9993$. By sim. triangles $AH+CG$ (37.748) : GH (3.3941) : : AH (24.7487) : $HI = 2.2253$; $AI = \sqrt{AH^2+HI^2} = \sqrt{24.7487^2+2.2253^2} = \sqrt{612.49815169 + 4.95196009} = \sqrt{617.45011178} = 24.8485$. Again, by sim. triangles, HI (2.2253) : HG (3.3941) : : AI (24.8485) : $AC = 37.8997 = BD$; now, by Problem V. Superficies, the area of the trapezium ABCD $= \frac{AC\times\overline{BF+FD}}{2} = \frac{AC\times BD}{2} = \frac{AC^2}{2} = \frac{37.8997^2}{2} = 718.1936$ *po.* = 4 *ac.* 1 *ro.* 38 *po.* Ans.

Ex. 13. Here, by Problem V. in Solids, $9^2\times.07958\times3 = 19.33794$ = content of the smaller rope; and $9^2\times.7854\times6 = 381.7044$ = content of the larger rope.

Hence 19.33794 : 381.7044 : : 22 *lbs.* : 434.25 *lbs.* Ans.

Ex. 14. Here half an acre = 2420 *sq. yds.*, and the area of a circle whose diameter is 1 = .7854.

Hence, as in Example 3, page 88, As .7854 : 2420 : : $1^2 : \frac{2420}{.7854} = 3081.2325$ = the square of the diameter.

Now $\frac{\sqrt{3081.2325}}{2} = \frac{55.5}{2} = 27.75$, the radius required.

Ex. 15. Here $7\frac{1}{4}\times2 = 14\frac{1}{2}$ *in.* = $\frac{29}{24}$ *ft.* = the difference of the diameters.

And $3\frac{1}{2}$ *ft.* + $14\frac{1}{2}$ *in.* = 4 *ft.* $8\frac{1}{2}$ *in.*, the diameter of the outside of the curb.

Hence $3\frac{1}{2}$ *ft.* + 4 *ft.* $8\frac{1}{2}$ *in.* = 8 *ft.* $2\frac{1}{2}$ *in.* = $8\frac{5}{24}$ *ft.* = $\frac{197}{24}$ *ft.* = sum of the diameters.

Wherefore, by Problem XV. page 82, $\frac{197}{24} \times \frac{29}{24} \times .7854 = \frac{4486.9902}{576} = \frac{747.8317}{96} =$ the area of the curb, and this multiplied by $8\,d. = \frac{747.8317}{12}\,d. = 62.3193\,d. = 5\,s.\ 2\frac{1}{4}\,d.$

Ex. 16. Here $10£. = 2400\,d.$, and $2\,s.\ 4\,d. = 28\,d.$

Hence $\frac{2400}{28} = \frac{600}{7} =$ the area of the semicircle in *sq. ft.*

And $\frac{600}{7} \times 2 = \frac{1200}{7} =$ the area of the whole circle.

Wherefore $.7854 : \frac{1200}{7} :: 1^2 : \frac{1200}{5.4978} = 218.26912583$ = the square of the diameter, the square root of which is 14.7739, the diameter required.

Ex. 17. Here the whole circle and each inner circle, after the several preceding rings are ground off, are as the numbers 7, 6, 5, 4, 3, 2, and 1; and the diameter of the whole circle is 60. Now, by Example 3, page 88, and by note at the bottom of page 162, we have

$$\left.\begin{array}{l}
\sqrt{7} : \sqrt{6} :: 60 : \frac{60\sqrt{6}}{\sqrt{7}} = \frac{60\sqrt{42}}{7} = 55.5492. \\
\sqrt{7} : \sqrt{5} :: 60 : \frac{60\sqrt{5}}{\sqrt{7}} = \frac{60\sqrt{35}}{7} = 50.7092. \\
\sqrt{7} : \sqrt{4} :: 60 : \frac{60\sqrt{4}}{\sqrt{7}} = \frac{120\sqrt{7}}{7} = 45.3557. \\
\sqrt{7} : \sqrt{3} :: 60 : \frac{60\sqrt{3}}{\sqrt{7}} = \frac{60\sqrt{21}}{7} = 39.2792. \\
\sqrt{7} : \sqrt{2} :: 60 : \frac{60\sqrt{2}}{\sqrt{7}} = \frac{60\sqrt{14}}{7} = 32.0713. \\
\sqrt{7} : \sqrt{1} :: 60 : \frac{60}{\sqrt{7}} = \frac{60\sqrt{7}}{7} = 22.6778.
\end{array}\right\}\ \text{The diameters of the stone after each preceding share has been ground off.}$$

Lastly, $60-55.5492=4.4508$, first share; $55.5492-50.7092=4.8400$, second share; and in like manner the others are found to be 5.3535, 6.0765, 7.2079, 9.3935, and 22.6778.

Ex. 18. Here $BD=\sqrt{\overline{DC}^2+\overline{BC}^2}=\sqrt{16400}=128.06248$.

$DF=DE=DC-CE=DC-CB=100-80=20$.

$BF=BD-DF=108.06248$.

$CL=BC-BL=BC-\frac{1}{2}BF=80-54.0312=25.9688$. Ans.

Note.—See the construction and figure at the bottom of page 226 in the work.

Ex. 19. Here 1 *ac.*=4840 *sq. yds.*, and, as in Example 14, page 224, $.7854:4840::1^2:\frac{4840}{.7854}=6162.464986=$ the square of the diameter; the square root of which is 78.5014, the diameter; and $\frac{78.5014}{2}=39.2507$, the length of the chord required.

Ex. 20. Here, by Example 4, page 163, and Note at the bottom of page 162, we have $\sqrt{1}:\sqrt{3}::16.5:16.5\sqrt{3}=28.5788$ *ft.*$=28$ *ft.* 7 *in.* Ans.

Ex. 22. Here, as in last Example, we have $\sqrt{1}:\sqrt{2}::26.3:26.3\sqrt{2}=37.19$. Ans.

Ex. 23. Here $\sqrt{39^2-15^2}=\sqrt{1296}=36$, the length of the standing part.

Hence $39+36=75$, the length of the pole required.

Ex. 24. By Problem XII. of Solids, the convex surface of a sphere whose diameter is 1 is 3.1416, and by Problem XIII. the solidity of the same sphere is .5236.

Now $\frac{3.1416}{.5236}=6$. Ans.

Ex. 25. Here $12^3 = 1728$, the number of cubic inches in a 12 inch cube, and $3^3 = 27$ do. in a 3 inch cube.

Hence $\frac{1728}{27} = 64$. Ans.

Ex. 26. Here $6^3 = 216 =$ the quantity borrowed, and $3^3 \times 2 = 54 =$ the part paid.

Now $\frac{216}{54} = 4$, wherefore he was paid $\frac{1}{4}$ part only.

Ex. 27. Here $\frac{64}{3.1416} = 20.3718 =$ the diameter of the base, half of which is 10.1859, the radius.

Hence $\sqrt{118^2 + 10.1859^2} = \sqrt{14027.75255881} = 118.4388$, the slant height of the cone.

Now, by Problem VI. of Solids, $\frac{118.4388 \times 64}{2} =$ 3790.0416 *ft.* = 421.1157 *sq. yds.*, the convex surface of the cone; which at 8 *d.* per *yd.*, amounts to 14£. 0 *s.* $8\frac{3}{4}$ *d.* + Ans.

Ex. 28. See the figure at page 162 of the work.

$BG = \frac{4 - 1.5}{2} = 1.25$; and $GC = \sqrt{BC^2 - BG^2} = \sqrt{8^2 - 1.25^2} = \sqrt{62.4375} = 7.9017$, the perpendicular altitude of the frustum.

Hence, by Problem IX. Solids, $(4^2 + 1.5^2 + 4 \times 1.5) \times .7854 \times \frac{7.9017}{3} = 24.25 \times .7854 \times 2.6339 = 50.165127705$ *ft.* the solidity of the frustum, which, at 12 *s.* per foot, amounts to 601.9815 *s.* = 30£. 1 *s.* $11\frac{3}{4}$ *d.*, the value required.

Ex. 29. Here $\sqrt{7.5} : \sqrt{8} :: 18.5 : \frac{18.5\sqrt{8}}{\sqrt{7.5}} = \frac{18.5\sqrt{60}}{7.5} = 19.1067$. Ans.

Ex. 30. Here 6 *ft.*=72 *in.*, and by Problem XII. of Solids, the convex surface$=72\times 72\times 3.1416=16286.0544$ *sq. in.*, which, at $3\frac{1}{2}$ *d.* per *inch*, amounts to 57001 *d.*= 237£. 10 *s.* 1 *d.*

Ex. 31. Here it is evident that the vessel required is to hold as much as one 46 *in.* in diameter, and twice 28, or 56 *in.* deep.

Hence $\sqrt{36} : \sqrt{56} :: 46 : \frac{46\sqrt{56}}{6}=57.37$. Ans.

Ex. 32. Here, by formula, page 186, the required difference $=\frac{(D^{\frac{3}{2}}-d^{\frac{3}{2}})^2}{D-d.}\times\frac{nh}{3}$, where D=4.23, *d*=3.7, *h* = 5.7, and *n*=.7854.

Wherefore $\frac{(D^{\frac{3}{2}}-d^{\frac{3}{2}})^2}{D-d.}\times\frac{nh}{3}=\frac{(\sqrt{4.23^3}-\sqrt{3.7^3})^2}{4.23-3.7}\times\frac{.7854\times 5.7}{3}=\frac{(8.6998-7.1171)^2}{.53}\times 1.49226=4.7263\times 1.49226=7.05$ *cubic inches.*

Note.—$D^{\frac{3}{2}}$ denotes the square root of D^3.

Ex. 33. Here, by Problem VIII. Solids, the solidity of the pyramid$=20^2\times\frac{150}{3}=400\times 50=20000$ *cubic feet.*

The tabular specific gravity, or the weight of a cubic foot of marble, is 2700 *oz.*, and of water, 1000 *oz.*

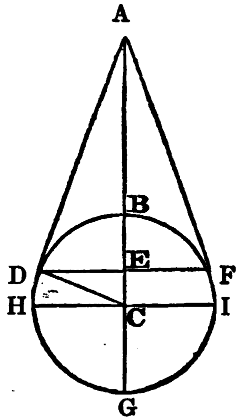
And by Problem II. Specific Gravity, As 1 *ft.* : 20000 *ft.* : : 2700 *oz.* : 54000000 *oz.* = $1506\frac{39}{56}$ *tons*, the whole weight; and $1000\times 20000=20000000$ *oz.*$=558\frac{2}{56}$ *tons*, the weight of the water displaced; their difference = $948\frac{37}{56}$ *tons*, the excess of the weight of the marble above that of the water, or the weight required to raise it to the surface.

Ex. 34. As in Example 2, page 161, $\sqrt[3]{3} : \sqrt[3]{1} :: 150 :$
$\frac{150\sqrt[3]{1}}{\sqrt[3]{3}} = {}^*\frac{150\sqrt[3]{9}}{3} = 50\sqrt[3]{9} = 104.0042,$

And $\sqrt[3]{3} : \sqrt[3]{2} :: 150 : \frac{150\sqrt[3]{2}}{\sqrt[3]{3}} = {}^*\frac{150\sqrt[3]{18}}{3} = 50\sqrt[3]{18}$
$= 131.0371.$

Now $150 - 131.0371 = 18.9629 =$ the length of the lowest part.
$131.0371 - 104.0042 = 27.0329 =$ do. middle do.
$104.0042 =$ do. upper do.

Ex. 35. Let BDGF represent the earth, C its centre, and B the place of a person on the surface. Draw the diameter BCG.



Now, since the curve surfaces of segments of a sphere are in proportion to their perpendicular altitudes, if BE *be taken equal* $\frac{1}{4}$BG, *and* DEF *be drawn at right angles to* BG, D *and* F *will be in the horizon of a person elevated*

* By multiplying the numerator and denominator of the last fraction by $\sqrt[3]{9}$.

so as to see one third of the earth's surface. Join DC, *and draw* DA *perpendicular to it, meeting* GB, *produced in* A, *which will be the place of the person.*

Hence (*cor.* 8.6) CE : CD : : CD : CA, that is $\frac{1}{2}$BG : $\frac{1}{2}$BG : : $\frac{1}{2}$BG : CA $=\frac{3BG}{2}$,

And AB=CA−CB$=\frac{3BG}{2}-\frac{BG}{2}=$BG; or the elevation of the person must be equal to the earth's diameter.

Ex. 36. Here 1 *cub. ft.* = 1728 *cub. in.*; and $\left(\frac{1}{40}\right)^2\times .7854=\frac{.7854}{1600}=\frac{.3927}{800}=$the area of one end of the wire.

Hence $1728\div\frac{.3927}{800}=\frac{1382400}{.3927}=3520244.4614$ *in.* = 97784.5684 *yds.*, the length required.

Ex. 37. Here $300\times200\times1=60000$ *cub. ft.*, the quantity of earth to be taken out of the ditch. And the length of the ditch is equal to the perimeter of the green, added to 4 times the width of the ditch$=(300+200)\times2+8\times4=$ 1032, which, being multiplied by $8=8256=$the area of the surface of the ditch.

Hence $\frac{60000}{8256}=7\frac{21}{86}$ *ft.*, the depth required.

Ex. 38. Here, by Problem I. Specific Gravity, 7425 : 384 : : 1728 : $\frac{663552}{7425}=\frac{24576}{275}$ the content of the ball in cubic inches. But the solidity of a sphere whose diameter is 1 *in.* is .5236.

Hence $.5236:\frac{24576}{275}::1^3:\frac{24576}{143.99}$, the cube of the diameter of the ball, and $\sqrt[3]{\frac{24576}{143.99}}=\sqrt[3]{170.678519342}=$

5.547=the diameter of the sphere, and 5.547+.1=5.647, the diameter of the bore required.

Ex. 39. Here $\frac{64+50}{2}\times 12=684$, the area of one side,

And $684\times 5=3420$=content in cubic feet.

Hence $\frac{3420}{128}=26\frac{23}{32}$ *cords*. Ans.

Ex. 40. Here $\frac{28}{12}=\frac{7}{3}$ *ft.* = the double thickness of the wall,

840 *links*=554.4 *ft.*=longer diameter within the wall,

$554.4+\frac{7}{3}=\frac{1670.2}{3}$ *ft.*=do. do. to the outside of the wall,

612 *links*=403.92 *ft.*=shorter diameter within the wall,

$403.92+\frac{7}{3}=\frac{1218.76}{3}$ *ft.*=do. do. to the outside of the wall.

Now, by Problem IV. Conic Sections, $554.4\times 403.92\times .7854 = 223933.248\times .7854 = 175877.1729792$ *sq. ft.* = 646 *per.*=4 *ac.* 0 *ro.* 6 *per.*, area enclosed.

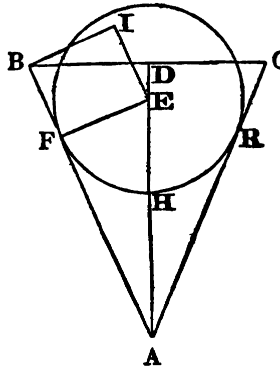
And $\frac{1670.2}{3}\times\frac{1218.76}{3}\times .7854=\frac{2035572.952}{9}\times .7854=\frac{1598738.9965008}{9}=177637.666$=area in *sq. ft.* to the outside of the wall.

175877.173 = area in square feet to the inside of the wall.

Their difference=1760.493=area in square feet that the wall stands upon.

Ex. 41. Let ABC be a vertical section of the glass, and AD the altitude. Draw BI perpendicular to BA, and equal to the radius of the ball (2). Draw IE parallel to BA, then will E be the place of the ball's centre. On

AB let fall the perpendicular EF, and with the centre E and radius EF describe the circle HFR representing a section of the ball.



Now $AB=\sqrt{AD^2+BD^2}=\sqrt{6^2+2.5^2}=\sqrt{42.25}=6.5$.

By sim. triangles BD(2.5) : BA(6.5) : : EF (2) : EA=5.2.

DE=AD−AE=6−5.2=.8.

DH=DE+EH=.8+2=2.8, the height of the segment immersed in water.

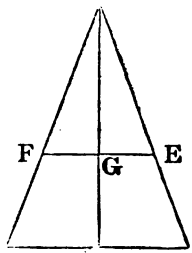
Now, by Problem XIV. Solids, the solidity of this segment $=(4\times 3-2.8\times 2)\times \overline{2.8}^2\times .5236=6.4\times 7.84+.5236$ $=26.2721536$. Ans.

Ex. 42. One cubic foot of water, by note, page 216, weighs 1000 *oz.* Avoirdupois. Hence the weight of the vessel=50000×1000=50000000 *oz.*=$1395\frac{5}{56}$ *tons*. Ans.

Ex. 43. Here 13 *gals.*=13×282 *cub. in.*=3666 *in.*, the content of the required kettle. Wherefore this question is the same as the last one in Miscellaneous Questions to Solids, page 44 of the Key.

Another method.

Let the frustum be completed to a cone, of which ABC is a section; AB and FE the diameters of the two ends of the frustum, and DC a line drawn from the apex to the middle of AB.



Now, by the question, $5 : 3 :: AB : FE$; but, by sim. $\triangle$'s, $AB : FE :: CD : CG$; wherefore (11.5) $5 : 3 :: CD : CG$, and (17.5) $5-3 : 3 :: CD-CG\ (GD=12) : CG = 18$; Hence $CD = CG+GD = 18+12 = 30$. Again, the solidities of similar cones being as the cubes of their altitudes, we have $CD^3 : CG^3 ::$ solidity of ABC : solidity FEC; and (17.5) $CD^3-CG^3\ (21168) : GC^3\ (5832) ::$ solidity of *ABEF (3666) : solidity of $FEC = \frac{5832 \times 3666}{21168} = 1010.0204$, which, added to ABEF $(3666) = 4676.0204$, the solidity of the cone ABC.

Now, the solidity of a cone, by Problem VIII. of Solids, is equal to the area of the base multiplied by one third of the perpendicular height; Hence the area of the base is equal to the solidity divided by one third of the perpendicular height $= 4676.0204 \div \frac{30}{3} = 467.60204$; there-

* The difference of the cones, ABC and FEC, or the given frustum.

fore the diameter $AB=\sqrt{\frac{467.60204}{.7854}}=\sqrt{595.3680163}=$ 24.4002, greater diameter; and $5:3::AB\ (24.4002):FE$ $=14.6401$, the less diameter.

Ex. 44. As in the last question, find the altitude (30) and solidity, 4676.0204, of the whole cone *ABC (see the last figure).

Then $4676.0204+3666=8342.0204$, the content of the cone, including the part added to the greater end. Now, since the solidities of cones are as the cubes of their altitudes, we have $4676.0204 : 8342.0204 :: 30^3 : 30^3 \times \frac{8342.0204}{4676.0204}$ and the cube root of this $=30\times\sqrt[3]{\frac{8342.0204}{4676.0204}}$ $=30\sqrt[3]{1.784}=30\times1.2128=36.384$, the altitude of the whole cone, including the extended part.

Hence $36.384-30=6.384$, the length of the part added.

* After finding the altitude, we may find the solidity of the cone by multiplying the square of the diameter of the greater end, as found in the first solution to the last question, by .7854, and this product again by $\frac{30}{3}$.

GAUGING.

PROBLEM XII.

PAGE 242.

To find the content of a cask of the first form.

Ex. 2. Here $(12^2+16^2\times 2)\times 20=656\times 20=13120$,
And $13120\times .0009\frac{1}{4}=12.136$ *ale galls.* }
Also $13120\times .0011\frac{1}{3}=14.869$ *wine galls.* } Ans.

PROBLEM XIII.

PAGE 244.

To find the content of a cask of the second form.

Ex. 2. Here $(12^2+16^2\times 2)-\frac{2}{5}$ of $(16-12)^2=656-6.4$ $=649.6$,
And $649.6\times 20\times .0009\frac{1}{4}=12.0176$ *ale galls.* }
Also $649.6\times 20\times .0011\frac{1}{3}=14.7243$ *wine galls.* } Ans.

PROBLEM XIV.

PAGE 245.

To find the content of a cask of the third form.

Ex. 2. Here $(16^2+12^2)\times 20=8000$,
And $8000\times .0014=11.2$ *ale galls.* }
Also $8000\times .0017=13.6$ *wine galls.* } Ans.

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188)104544(556
    940
    ----
    1054
     940
    ----
    1144
    1128
    ----
      16
```

556 tons. Ans.

THE END.